HEARING OVERCOMES UNCERTAINTY RELATION

AND MEASURES DURATION OF ULTRASHORT PULSES

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The human hearing sense is an astonishingly effective signal processor. Recent experiments [1, 2, 3] suggest that it is even capable of overcoming limitations implied by the time-frequency uncertainty relation. The latter, mostly known from quantum mechanics, requires that the product of uncertainties in time and frequency, $\Delta t \cdot \Delta f$, cannot be smaller than the limiting value $\Delta t \cdot \Delta f = (1/4\pi)$, which holds when the signal is a harmonically oscillating function with a Gaussian envelope.

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The acoustic pressure of such a signal is $p(t)=p_0\exp(-t^2/2(\Delta t)^2)\cos(2\pi f_1 t)$, where f_1 is the underlying frequency. The spectrum of the signal is also Gaussian, $p(f)=p_0\exp(-(f-f_1)^2/2(\Delta f)^2)$ and the widths of the two Gaussian functions are inversely proportional to one another.

Whereas the theorem follows directly from properties of the Fourier transform, its significance both in quantum mechanics and in macroscopic physics is still debated. The problem resides in the meaning of words such as "width", "extent" or "uncertainty" in the theory of signal processing and in statistics. According to the standard quantum mechanical interpretation a sufficiently long measurement time is needed to achieve a prescribed precision in determination of the energy $E=hf$ of a stationary quantum state. Shortening the duration time of a pulse leads to broadening of its spectrum. Broadband properties of attosecond optical pulses are now investigated for applications in condensed matter studies [4]. Here we propose some simple acoustic tests that help to elucidate the extraordinary abilities of the human hearing when exposed to pulses of duration times comparable to or shorter than one oscillation cycle. This is what we call ultrashort pulses.

The ear is a frequency detector. In particular, a periodic acoustic signal of frequency f produces a sensation of pitch H (the tone height). A known tune, *i.e.*, a sequence of sounds of different pitches, is recognizable as long as the frequency ratios of the consecutive sounds are conserved. This impressive equivalence of pitch differences – called musical intervals – to frequency ratios implies that the pitch-frequency relation is strictly logarithmic. For example, in the conventions adopted by MIDI (Musical Instrument Digital Interface) the relation reads $H=69+12\log_2(f/440\text{Hz})$. The exact evolutional and physiological foundations of this relation are not well understood to the best of our knowledge.

Among all periodic signals the purely sinusoidal ones, called pure tones, are the closest counterparts of stationary quantum states. They are mostly exploited in what follows. The presence of higher harmonics, which does not affect the periodicity of the signal, results in changes in what is called timbre (tone colour) of the sound.

Sinusoidal signals with Gaussian envelopes

The test ep220_cos.mp4 [t1] is designed to allow the reader to appreciate the effect of a Gaussian envelope on the perceived pitch of a pure tone. The listener is provided with three sounds: i) a relatively long reference tone, ii) a cosine signal $\cos(2\pi f_1 t)$ wrapped in a Gaussian envelope $\exp(-t^2/2(\Delta t)^2)$ and iii) the envelope alone. The fundamental frequency is f_1=220Hz and the width Δt of the envelope decreases in the consecutive sequences in the range from 22.7 ms to 0.0227 ms. Detailed instructions for the test are supplied in Box 1.

The results obtained for three test persons (three of the authors) are shown in Fig. 1. All persons notice an increase in the perceived pitch f_p with decreasing envelope width Δt. The pitch of the sole envelope f_{EP} (Envelope Pitch) starts to be noticeable if Δt is less than about 1.13ms. The test persons report the two to be equal, $f_{EP}=f_p$, for extremely short pulses, Δt=0.159ms. We invite the reader to listen to the test ep220_cosRETRO.mp4 [t1] to appreciate the effect at a different aspect. The fact that the effective perceived pitch f_p of a short signal is higher than the pitch corresponding to the frequency f_1 of the underlying cosine indicates how our sense of hearing overcomes the uncertainty principle. The time needed to determine a higher frequency is, put simply, shorter. Now we have to find a mechanism to explain this rise in pitch. A hint is visible in Fig. 1 where a significant number of results concentrate around the frequency $f_p = 3\times f_1$, *i.e.*, that of the third harmonic. The signal seems, therefore, to be nonlinearly processed (distorted) prior to reaching the heart of the detector, *i.e.*, the basilar membrane of the cochlea. The nonlinear distortion may take place in the ear itself as well as outside. Another reason for the increase in the pitch, this time of purely linear origin, can be related to the broadening of the spectrum with decreasing duration, so that some higher frequencies, not necessarily harmonic, are also presented to the ear. In any case, the ratio f_p/f_1 is an estimate of the factor by which the uncertainty relation is "beaten" by our hearing organs. The factor is roughly constant in the region where the supposed third

BOX 1: TEST FOR THE EFFECT OF GAUSSIAN ENVELOPE ON THE PITCH OF A PURE TONE

In the test en220_cos.mp4 the listener is asked to determine the pitches of three consecutive sounds. The first reference sound is a long cosine signal so that its pitch is readily recognizable. The second sound is a cosine signal $\cos(2\pi f_1 t)$ wrapped in a Gaussian envelope $\exp(-t^2/2(\Delta t)^2)$. The width Δt of the Gaussian decreases in the consecutive sequences. The third sound is the Gaussian envelope alone. The signal underlying frequency is f_1=220Hz. The reference sound has a frequency $3\times f_1$ (third harmonic of the signal). The width Δt decreases with time of the experiment in the following manner: i) starting from $1000/F_s$=22.676 ms to $200/F_s$=4.535 ms by $50/F_s$=1.135 ms and: ii) from $199/F_s$= 4.533 ms to $1/F_s$=0.0227 ms by $1/F_s$ where F_s=44100 s^{-1} is the standard sampling rate in the *.wav format of sound files. The listener sees the number of the experimental sequence and the corresponding width of the envelope in ms on the screen. This allows one to trace curves analogous to Fig. 1. The test Env220_cosRETRO.mp4 contains the same sounds in order of decreasing Δt.

BOX 2: TEST FOR EFFECTIVE ENVELOPE PITCH OF A GAUSSIAN PULSE

The test eep.mp4 consists of pairs of sounds. The first sound is a cosine wave of frequency 660 s^{-1}, long enough to easily determine its pitch. The second sound is a pure Gaussian envelope $\exp(-t^2/2(\Delta t)^2)$. The maximum amplitudes of the signals are equal. The width Δt of the Gaussian decreases with the number of pairs in the manner analogous to Box 1 i) starting from $1000/F_s$ to $200/F_s$ by $50/F_s$ and ii) from $199/F_s$ to $1/F_s$ by $1/F_s$, where F_s=44100s^{-1} is the standard sampling rate The sequence number and the width in ms are visible on the screen. The listener is asked to attribute an interval to each pair of sounds heard. The corresponding frequency $f_{EP}(\sigma)$ should be calculated knowing the frequency ratios defining each interval (see, *e.g.*, http://en.wikipedia.org/wiki/List_of_pitch_intervals). Listeners with absolute pitch hearing may find the reference sound redundant. The listeners without knowledge of musical intervals may give their responses in terms of beginnings of known tunes. Multiple listening to the test may give more precise results. Sometimes subjects hesitate to attribute a perceived interval due to the approximate character of the Effective Envelope Pitch. The responses may also be different for subjects who are accustomed to different intervals than those used in European music. The file eepRETRO.mp4 presents the sequence in order of increasing Δt.

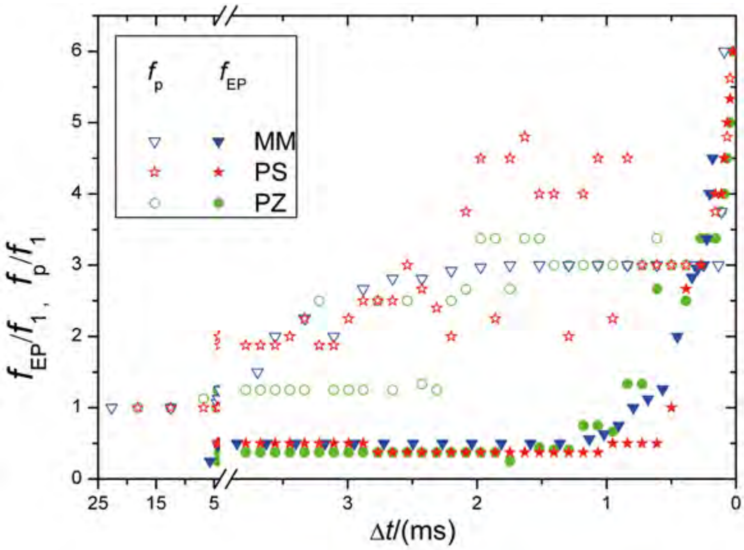
harmonic is generated and, at the same time, the pulse is too short to allow the fundamental frequency to be detected. A significant increase in the factor is visible for extremely short pulses. Noteworthy is that the "beating of the uncertainty principle" by more than a factor of ten, as reported in [2], concerns essentially quantities which are of a very different nature than the pulse's duration time Δt.

The shorter the sharper

An intriguing result of the test discussed above is that a certain pitch is attributed to a purely Gaussian pulse without any underlying periodic signal. The pulse must be very short for this to be the case, *i.e.*, Δt<1.5ms. One may realize that such pulses are in fact clicks or snaps. In other words: they are point-like events on the axis of time. The shape and the duration of such pulses are surely beyond the reach of human perception. Yet differences in what can be qualified as pitch or timbre offer the possibility to obtain insight into such properties. Let us have a closer look at the phenomenon.

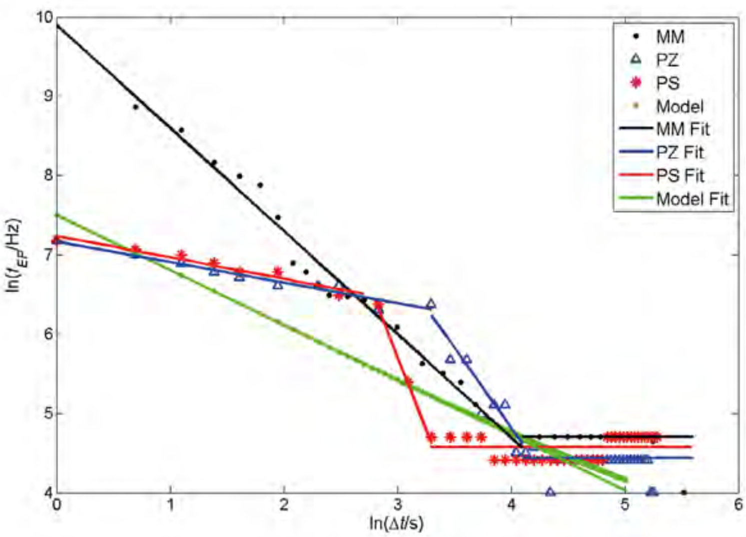
The dependence of the frequency f_{EP} (the perceived pitch of the envelope alone) on the pulse's width Δt can be studied with the test eep.mp4 [t2] described in Box 2. The listener is asked to determine the musical interval between a relatively long reference tone and the pulse. Knowing the intervals in terms of the frequency ratios one can trace the desired function $f_{EP}(\Delta t)$. It is clear that the shorter pulses appear sharper. The results obtained by the authors are represented in Fig. 2 as a log-log plot.

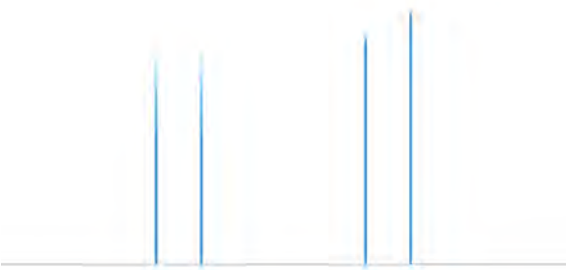
The incontestable straight-line behaviour indicates a power law $f_{EP}(\Delta t)\cong 1/\Delta t^{\alpha}$. It is interesting that the exponent α is almost identical for the two test persons who lack an absolute pitch hearing (α=0.271±0.031 for PS and α=0.260±0.023 for PZ) and much higher for the only test person who does have absolute pitch hearing



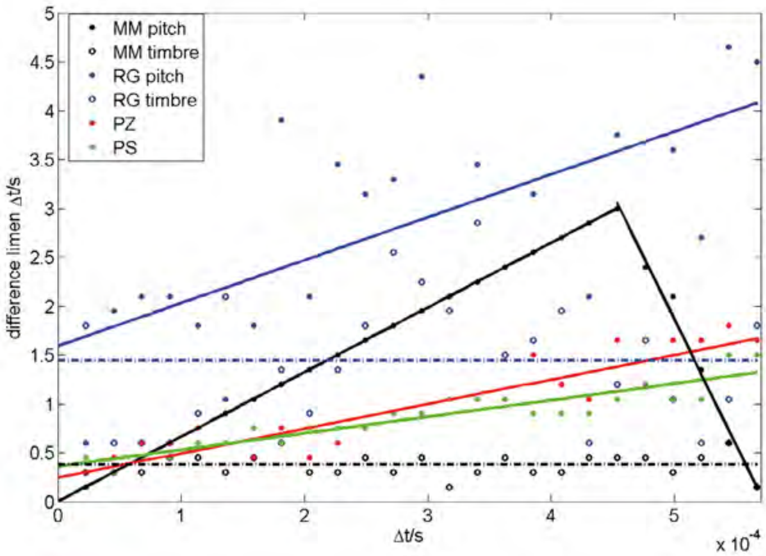
▲ **FIG. 1:** Effective pitch of a cosine signal with a Gaussian envelope and the pitch attributed to the envelope itself as a function of the pulse's duration Δt, for three different test persons (MM, PS and PZ). Here, f_1 is the frequency of the enveloped cosine signal, f_p the perceived pitch of the enveloped cosine signal and f_{EP} (the envelope pitch) the perceived pitch of the envelope itself.

▼ **FIG. 2:** Pitch attributed to purely Gaussian pulses (envelopes only) as a function of duration Δt.





▼ **FIG. 3:** Assessment of the width of two narrow lines of equal area, using their height as a key.



▶ **FIG. 4:** Just noticeable difference in pulse width *vs.* pulse width as detected by the authors (MM, RG, PZ and PS). The lines represent linear fits.

(α=1.30±0.05 for MM). A plausible mechanism of the sensation of the envelope pitch is that the spectrum of the pulse is filtered by the ear in such a way that the maximum of the filtered spectrum shifts towards higher frequencies with increasing width Δf of the incoming spectrum, *i.e.*, with decreasing duration Δt. To model such a filter we adopted the idea of Helmholtz that an incoming signal excites a series of damped oscillators, each of which tuned to its own distinct *eigenfrequency*. The effective pitch corresponds to the oscillator that, when excited by the pulse, attains the highest amplitude. More details on the model can be found in [3]. The frequency of the most excited oscillator is also shown in Fig. 2. The power law is satisfied to great accuracy in the region studied, but the exponent is still different: α_{theory}=0.6939±0.0005.

Sensing submillisecond time intervals

It is clear that the increase in the effective pitch with decreasing duration Δt is most significant for the shortest pulses. This follows, of course, from the uncertainty principle for Gaussian envelopes: $\Delta f=1/(4\pi\Delta t)$. Therefore, paradoxically, changes in the duration are most easily discernible for the shortest pulses. A similar effect occurs in the assessment of the width of a line drawn on a paper as depicted in Fig. 3. It is not easy to detect a difference in width of the two vertical lines on the left hand side. The difference is, however, immediately clear if the area covered by each line, *i.e.*, the product of its width and length, is known to be constant, as is seen on the right-hand side.

A practical measurement of the duration of a Gaussian pulse with the use of its spectral properties raises the question of the discriminability – also known as the just noticeable difference – $d_l\Delta t$ (difference limen) of duration times. With the tests described in Box 3 [t3] the reader will find for herself/himself the smallest noticeable difference $d_l\Delta t$ as a function of Δt. The experience of the authors suggests that, when provided with two short purely Gaussian pulses of slightly different widths, some people distinguish two qualities: the effective pitch and the timbre. More precisely, if the duration times are very close, some subjects judge the pitches equal although they still notice a difference, which can be qualified as a difference in timbre. The results obtained by the authors indicate that the 'difference limen' associated with the effective pitch is fairly proportional to the duration of the pulse. Thus, it follows the Weber-Fechner law [5] stating that $d_l\Delta t/\Delta t=const.$ In contrast, the difference limen as judged by the timbre

BOX 3: TEST FOR DISCRIMINATION LIMEN OF DURATION TIMES OF GAUSSIAN PULSES

The reader can check the discrimination limens for the width parameter Δt of Gaussian signals exp($-t^2/2(\Delta t)^2$) in the range $1F_s<\Delta t<25/F_s$, *i.e.*, 0.023ms<Δt<0.567ms, where F_s=44100s^{-1} is the standard sampling rate. There are 25 files here, each corresponding to the initial width parameter $\Delta t=NN/F_s$ where the integer number NN is indicated in the file name dlNN.mp4. Each file contains a series of pairs of sounds. The first sound has width parameter $\Delta t=NN/F_s$ and the second $(NN+n\cdot d\Delta t)/F_s$, where $d\Delta t=0.15/F_s$. The listener is asked to indicate the pair number n_l for which the sounds start to appear different. The corresponding difference $n_l\cdot d\Delta t/F_s=d_l\Delta t$ is an estimate of the discrimination limen sought. The Weber-Fechner law is satisfied if the discrimination limen is proportional to the initial width $\Delta t=NN/F_s$.

turns out almost independent of Δt, *i.e.*, $d_l \Delta t = const$. Two of us noticed just a difference without paying attention to the nature of the difference. The results were then just linear combinations of both behaviours as seen in Fig. 4. The values of the constants depend on the subject. The readers may try to trace their own curves.

In conclusion, simple acoustic tests are able to reveal mechanisms which make the ear overcome the uncertainty relation. The same mechanisms allow a human subject to distinguish the duration times of very short acoustic pulses provided that the shapes of the pulses are well defined. This phenomenon may be useful in designing devices aimed at measuring parameters of ultrashort pulses. ■

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Links to tests

[t1] https://drive.google.com/folderview?id=0BwQPqgssghTtZDNwY2Z0NmgzRVE&usp=sharing

[t2] https://drive.google.com/folderview?id=0BwQPqgssghTtQkhHanZJd3hScDg&usp=sharing

[t3] https://drive.google.com/folderview?id=0BwQPqgssghTtZUdJOU9XX0pna00&usp=sharing