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Beryllium neutron activation counter for pulsed D-D fusion sources

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Abstract

The fusion reaction occurring in DD plasma is followed by emission of 2.45 MeV neutrons, which carry out information about fusion reaction rate and plasma parameters and properties as well. Neutron activation of beryllium has been chosen for detection of DD fusion neutrons. The cross-section for reaction ^{9}Be (n, α) ^{6}He has a useful threshold near 1 MeV, which means that undesirable multiply-scattered neutrons do not undergo that reaction and therefore are not recorded. The product of the reaction, ^{6}He, decays with half-life $T_{1/2}$ = 0.807 s emitting β^{-} particles which are easy to measure.

Large area gas sealed proportional detector has been chosen as a counter of β^- particles which leave activated beryllium plate. The plate with optimized dimensions adjoins the proportional counter entrance window. Such set-up is also equipped with appropriate electronic components and forms beryllium neutron activation counter.

The density of neutron flux on beryllium plate can be determined from the number of counts. Therefore, a proper calibration procedure needs to be performed to establish such a relation. The measurements with the use of known β^- source have been done. In order to determine the detector response function such experiment has been modelled by means of MCNP5 – the Monte Carlo transport code. It has allowed a proper application of results of the transport calculations of β^- particles emitted from the radioactive ^6He and reaching the proportional detector active volume.

In order to test the counter system and measuring procedure a number of experiments have been performed on PF-6, PF-1000 and PF-4 devices. The experimental conditions have been simulated by means of MCNP5. The correctness of simulation outcome have been proved by measurements with known radioactive neutron source. The results of the DD fusion neutrons measurements have been compared with other neutron diagnostics.

1. Introduction

Plasma-Focus (PF) devices belong to the family of the dynamic, non-cylindrical Z-pinches and are based on pulsed high-current discharges between two coaxial electrodes placed inside a working gas (usually pure deuterium). In general, the PF devices can be considered as a power transformer in which energy stored in the magnetic field is abruptly converted into energy of the pinch plasma. The propagation period, from the breakdown to the pinch formation, lasts usually a few microseconds. The final stages of PF discharges are much shorter and they last from several tens to few hundreds of nanoseconds (depending on the PF device scale).

The essential problem to be solved in PF studies is connected with the understanding of physics which dominates the deuterium plasma formation. This question is closely related to neutron production mechanisms, plasma dynamics and physics of the conversion of magnetic field energy into pinch plasma energy. A realization of such problems requires preparation of a suitable diagnostics system.

For a case of deuterium as a working gas there are two equally possible nuclear fusion reactions:

$$^{2}D + {}^{2}D \rightarrow {}^{3}He(0.82\ MeV) + n(2.45MeV) + 3.27\ MeV$$
$$^{2}D + {}^{2}D \rightarrow {}^{3}T(1.01\ MeV) + {}^{1}H(3.03\ MeV) + 4.03\ MeV$$

Neutrons with the energy of 2.45 MeV are the products of 50% fusion events.

The most important part in designing DD fusion neutron counter is the selection of activation material which should meet certain criteria:

1. Nuclides produced from the reaction with neutrons should emit radiation which is easy to measure.
2. Cross-section of the reaction of neutrons with the activation material should be high enough to make possible registration of the products of the reaction.
3. The half-life of the products of the reaction with neutrons should be short enough to be useful to work with DPF devices which repetitive rate is of the order of 1 Hz.

It has been chosen to measure β^- particles since they are easy for detection. Moreover β^- decay nuclides have half-lifes within the limits fulfilling our requirements.

Beryllium with only one naturally occurring isotope ^{9}Be has been chosen as an activation material. The following reaction is taken into account:

$$^{9}Be\ (n, \alpha)\ {}^{6}He \rightarrow {}^{6}Li + \beta^-$$

The cross-section for that reaction is shown in Fig. 1. It has a useful threshold near 1 MeV, which means that undesirable scattered neutrons do not undergo that reaction and therefore are not measured. The cross-section for 2.45 MeV neutrons is about 80 mbarn and is sufficient for our purpose. The product of the reaction, ^{6}He, decays with half-life $T_{1/2}$ = 0.807 s emitting β^{-} particles with mean energy of 1.578 MeV and maximum energy of 3.508 MeV.

The thickness of the beryllium activation plate should be determined by mean range of electrons coming from the decay of obtained nuclide (^{6}He). It is about 5 mm for the β^{-} particles having mean energy in beryllium. Taking also into account mechanical properties of beryllium, 2 mm thickness has been chosen for the beryllium activation plate.

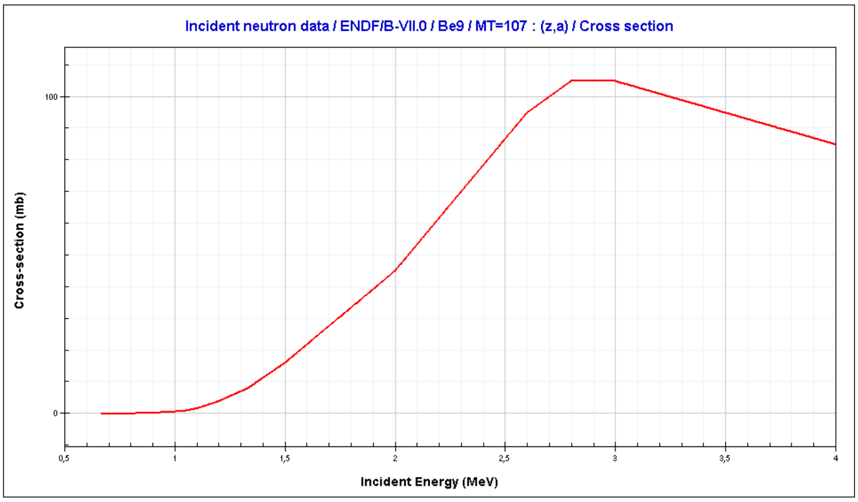

Fig. 1. Cross-section for the reaction $^{9}Be(n,\alpha)^{6}He$ from ENDF/B-VII.0

2. Measuring set

Large area gas sealed proportional detector SP-126C made by Canberra has been chosen as β^{-} particles counter. The active area is 126 cm^2 (113 mm × 113 mm), the window is made of titanium with surface density of 4.5 mg/cm^2 and the filling gas is P-10 (90% argon, 10% methane). The window is supported by a steel supporting mesh. Beryllium plate (GoodFellow made, 99.0% purity), with dimensions 100 x100 x 2 mm, 36 g adjoins the steel mesh and is centered in relation to the window. Complete detector can be seen in Fig.2.

Fig. 2. Beryllium neutron activation detector

The energy (W) required to form one ion pair in P-10 is equal to 26 eV [1]. According to performed measurements the gas multiplication factor (η) of the SP-126C detector (input voltage 1650 V) has been established to be about $1 \cdot 10^3$.

Two spectrometric sets (BC-W and BC-K) consisting of the beryllium activation counter have been completed. The BC-W counter is prepared to be implemented on PF-1000 device operated by IPPLM, Warsaw, whereas, the BC-K set is currently installed at the PF-4 device operated by IFJ, Kraków. Both counters are assigned as a permanent neutron yield monitors. Before ultimate installation the counters (and measuring procedure) have been tested in a series of experiments.

The BC-K is equipped with the following Canberra made sub-assemblies:

- high voltage power supply model 3102D;
- preamplifier model 2006 with amplification (ξ) equal to $4.7 \cdot 10^{-8}$ V/ion pair;
- spectroscopy amplifier model 2022 with amplification (κ) set to 279;
- dual counter/timer model 512.

Because the input threshold (U_p) of used counter model 512 is set to be 1.5 V, therefore the threshold energy deposited in the BC-K detector to create the pulse is:

$$E_p = \frac{W \cdot U_p}{\eta \cdot \xi \cdot \kappa} \approx 3\text{keV}. \qquad (1)$$

The BC-W spectrometric set is equipped with two separate channels for proportional counters, battery operated preamplifiers Amptek A121 (sensitivity p $\approx 10^5$ electrons) with fiber optics links to the computer, HV power supply (two independent gating times for both channels) and data acquisition system based on Advantech PCI-1780U 8-channel, 16-bit counter/timer universal PCI card. The threshold energy deposited in BC-W detector to produce the pulse is:

$$E_p = \frac{W \cdot p}{\eta} \approx 3\text{keV}. \qquad (2)$$

In the case of BC-W spectrometric set a counting delay of 150 ms following the pulse has been applied. After a number of experiments the delay has been reduced to 50 ms. Such value allows to avoid direct registration of gammas and neutrons from plasma. The total counting time has been set to 2.44 sec (three times ^{6}He half-life). In the case of the BC-K assembly the delay value has varied between about 200 and 500 ms due to the lack of automatic triggering system. The delay value has been taken into account to correct numbers of pulses recorded by the counters.

3. Calibration of the beryllium neutron activation counter

The idea of calibration of the proportional counter as a β^- particles detector is discussed in this chapter. The calibration of the beryllium neutron activation counter consists also of neutron transport calculations which are specific to particular experimental conditions. Certain cases of such calculations connected with performed experiments are described in a further part of this report. Obtained calibration coefficients for various experiments are provided there as well.

Calibration of the beryllium neutron activation counter requires measurements with the use of calibration β^- source. Flat $^{90}Sr/^{90}Y$ calibration source consisting of 0.5 mm thick aluminium plate (100 mm x 150 mm) covered by thin film of lacquer (polymethacrylate) containing ^{90}Sr has been chosen. Monte Carlo calculations of β^- particles transport, neutrons transport and beryllium activation by neutrons are necessary to complete the calibration. The MCNP5 [2] Monte Carlo code with MCNP5DATA [3] cross section library have been used for the above mentioned calculations.

It has been essential to prepare whole Canberra SP-126C proportional detector geometrical model. Due to the lack of the information about details of the construction of the detector, the number of X-ray pictures of it have been taken. It has allowed roughly estimating the volume of the filling gas and the thickness of the walls of the case. Cut view of the geometrical model of the counter is presented in Fig. 3.

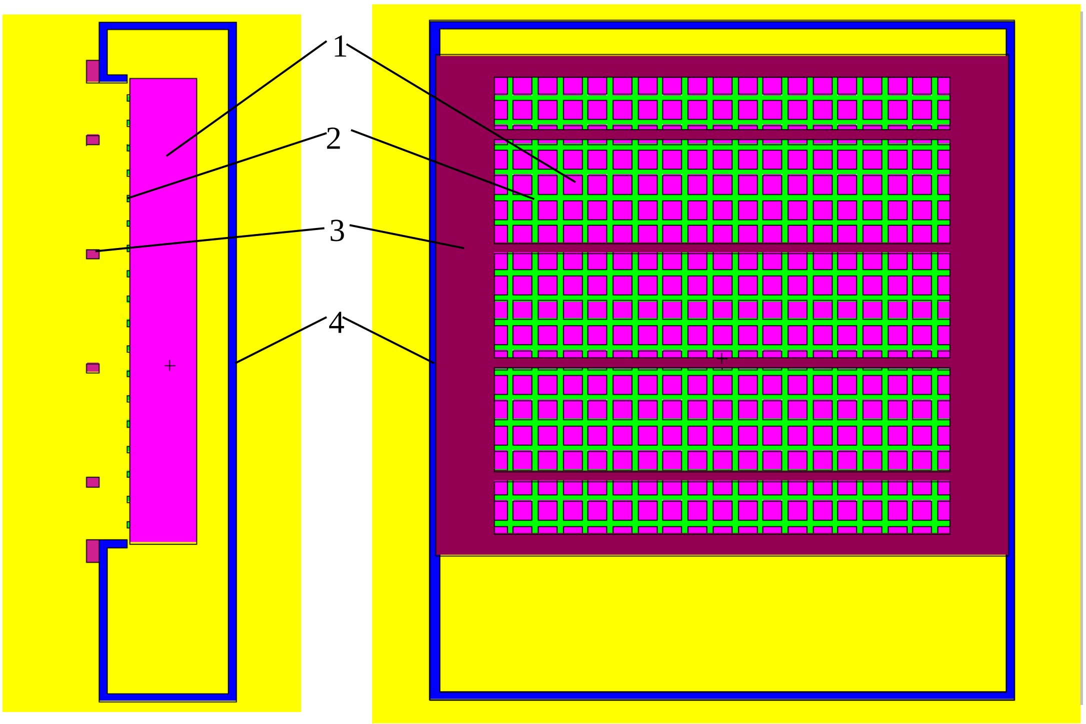

Fig. 3. Cut view of the geometrical model of the SP-126C proportional counter from MCN.
1 – filling gas P-10, 2 – supporting steel mesh, 3 – duralumin protecting mesh, 4 – titanium alloy case, 0.01 mm titanium foil wraps filling gas and cannot be seen on the picture.

As it was mentioned before, fast neutrons induce in the beryllium plate the nuclear reaction $^{9}Be(n,\alpha)^{6}He$, which product decays with the half-life $T_{1/2}$ = 0.807 s by emitting β^{-} particles, which are recorded by the proportional detector located adjacent to the beryllium plate. The beryllium plate adjoining the detector has been simulated by means of MCNP BC-input as a uniform volume source of beta-particles coming from decay of ^{6}He. As a result of the calculations the β^{-} particles energy spectrum inside the filing gas has been obtained.

Based on such a simplified geometry of the proportional detector two computational MCNP inputs have been prepared. BC-input consists of the SP-126C geometry with beryllium plate adjacent to the steel supporting mesh. CAL-input consists of the SP-126C geometry and the geometry of a $^{90}Sr/^{90}Y$ calibration source adjacent to the duralumin protecting mesh.

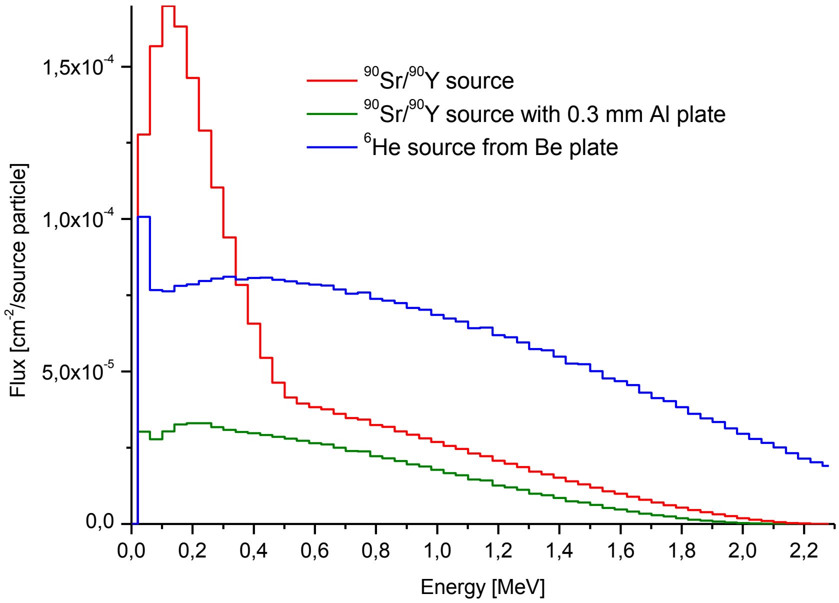

Fig. 3. The β^{-} particle energy spectrum inside the proportional counter.

According to the above mentioned MCNP calculations a flat $^{90}Sr/^{90}Y$ calibration source has been covered by the 0.3 mm thick aluminium plate (100 mm x 100 mm). Excess active parts of the source have been shielded by 1.3 mm thick aluminium layer. That shield has been also implemented to the CAL-input. As a result of such experiment the number of counts per source activity has been obtained.

Due to the applied settings of the proportional detector amplifier, preamplifier and counter (see above) the minimal energy which particle should deposit in the detector filing gas volume to be recorded is equal to 3 keV. Therefore, the number of β^{-} particles which deposit in SP-126C detector at least 3 keV has been calculated by means of MCNP in the case of both experiments: the calibration with $^{90}Sr/^{90}Y$ source and measurements with ^{6}He source from beryllium plate. The simulations have resulted in $5.167\cdot10^{-2} \pm 0.007\cdot10^{-2}$ of the β^{-}

particles from $^{90}Sr/^{90}Y$ source shielded by aluminium plate (CAL-input) which deposit at least 3 keV and 0.20825 ± 0.00006 in case of ^{6}He source from beryllium plate (BC-input).

One can compare results of the above mentioned simulations with the measured number of counts per calibration source activity. Such value is the so called calculation to experiment ratio (*C/E*). As a result of the measurements using BC-W detector $5.49 \cdot 10^{-3} \pm 0.05 \cdot 10^{-3}$ counts have been recorded per one β^- particle emitted from the source, and $4.22 \cdot 10^{-2} \pm 0.03 \cdot 10^{-2}$ in the case of BC-K detector. Therefore, calculation to experiment ratio *C/E* = 9.4 ± 0.1 in the case of the BC-W and *C/E* = 1.22 ± 0.02 in the case of BC-K. The ratio is inversely proportional to the detector geometry efficiency and is used in further calculations.

The next step is an MCNP simulation of the beryllium response to the particular neutron source in a defined measuring geometry.
Then the number of pulses recorded by the beryllium activation counter is:

$$N_{Be}^{\exp} = \frac{N_{Be}^{calc}}{C/E} \cdot N_{at} \cdot \langle \sigma, \varphi \rangle_n \cdot Y_n , \tag{3}$$

where N_{Be}^{calc} is the number of pulses created in the detector working gas volume per one beta particle from ^{6}He decay (MCNP calculations); N_{at} is the number of beryllium atoms; $\langle \sigma, \varphi \rangle_n$ is the rate of $^{9}Be(n,\alpha)^{6}He$ reaction per one source neutron (from the MCNP calculations); Y_n is the source total neutron yield.

According to the MCNP calculations for the geometry applied in both spectrometric sets the relative number of beta particles from beryllium plate which deposit at least 3 keV in the proportional counter working gas is N_{Be}^{calc}= 0.20825 ± 0.00006.

4. Neutron yield measurements performed by means of beryllium activation counters

Based on Eq.(3) the calibration coefficient (*K*) of beryllium activation counter is defined as follows:

$$K = \frac{C/E}{N_{Be}^{calc} \cdot N_{at} \cdot \langle \sigma, \varphi \rangle_n} , \tag{4}$$

and then:

$$Y_n = N_{Be}^{\exp} \cdot K , \tag{5}$$

where *C/E* is calculation to experimental ratio of the number of β-particles causing the pulse in the counter; N_{Be}^{calc} is the number of pulses created in the detector working gas volume per one beta particle from ^{6}He decay (MCNP calculations); N_{at} is the number of beryllium atoms; $\langle \sigma, \varphi \rangle_n$ is the rate of $^{9}Be(n,\alpha)^{6}He$ reaction per one source neutron (from MCNP calculations); Y_n is the source total neutron yield.

In order to test the counter systems and measuring procedure a number of experiments have been performed on PF-6, PF-1000 and PF-4 devices. The experimental conditions have been simulated by means of MCNP (see the following chapters). The rate of $^{9}Be(n,\alpha)^{6}He$ reaction per one source neutron has been, therefore, obtained in each case allowing calculation of the calibration coefficients. The ENDF/B-VII.0 nuclear cross-section library has been implemented to obtain the $^{9}Be(n,\alpha)^{6}He$ reaction rate. The results of the simulations are presented in Tab. 1. Due to uncertainty of the plasma neutron source position the simulations of PF-4 device have been performed for a number of point D-D sources located at various distances from the central electrode (the anode).

Table. 1. $^{9}Be(n,\alpha)^{6}He$ *reaction rates* $\langle \sigma, \varphi \rangle_n$ *calculated by means of MCNP and calibration coefficients K for various experimental arrangements.*

Counter model	$\langle \sigma, \varphi \rangle_n$	K
BC-W (PF-1000)	$9.65\cdot10^{-32} \pm 0.01\cdot10^{-32}$	$2.05\cdot10^{8} \pm 0.09\cdot10^{8}$
BC-K (PF-6)	$2.57\cdot10^{-30} \pm 0.01\cdot10^{-30}$	$9.01\cdot10^{7} \pm 0.36\cdot10^{7}$
BC-K (PF-4) 0 cm	$3.14\cdot10^{-29} \pm 0.02\cdot10^{-29}$	$7.83\cdot10^{4} \pm 0.39\cdot10^{4}$
BC-K (PF-4) 1 cm	$3.35\cdot10^{-29} \pm 0.02\cdot10^{-29}$	$7.33\cdot10^{4} \pm 0.37\cdot10^{4}$
BC-K (PF-4) 2 cm	$3.22\cdot10^{-29} \pm 0.02\cdot10^{-29}$	$7.63\cdot10^{4} \pm 0.38\cdot10^{4}$
BC-K (PF-4) 3 cm	$3.07\cdot10^{-29} \pm 0.02\cdot10^{-29}$	$8.01\cdot10^{4} \pm 0.40\cdot10^{4}$
BC-K (PF-4) 4 cm	$2.92\cdot10^{-29} \pm 0.01\cdot10^{-29}$	$8.43\cdot10^{4} \pm 0.38\cdot10^{4}$
BC-K (PF-4) Pu-Be source (Bi)	$1.75\cdot10^{-29} \pm 0.01\cdot10^{-29}$	$1.41\cdot10^{5} \pm 0.07\cdot10^{5}$

4.1. Measurements with BC-K at the PF-6 device

PF-6 plasma focus operated at IPPLM, Warsaw, is equipped with a 28 μF condenser bank and a special geometry chamber (∅ 55 mm) designed as the efficient pulse source of neutrons (up to 10^{9} per shot). Charging voltage was 15-17 kV and deuterium pressure in the chamber was 15-19.5 Tr.

Based on a number of experiments it has been decided to put the BC-K counter inside the Faraday cage in order to avoid the influence of electromagnetic disturbances generated by the PF-6 device. The cage with the counter has been placed 0.45 m from the PF-6 chamber. Based on already existing a very simplified MCNP PF-6 input (see Fig. 4) such geometry with point D-D fusion neutron source has been reconstructed allowing calculation of the reaction rate and, thus, the calibration coefficient. No attempt has been made to make detailed geometry input since that experiment has been intended as a rough check of the method.

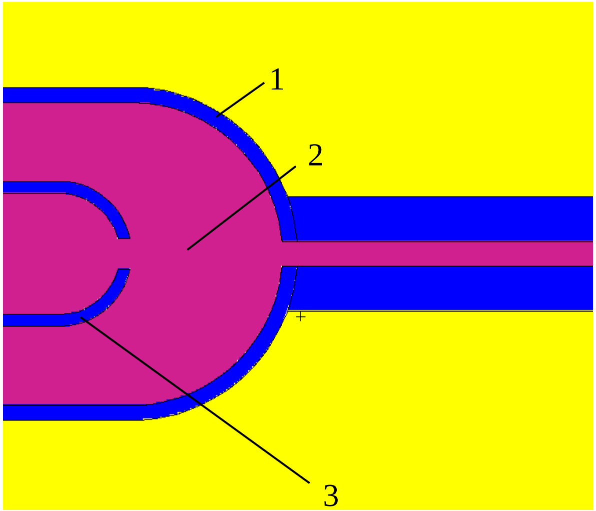

Fig. 4. Simplified MCNP geometry input of PF-6 device.
1 – copper vacuum chamber and the cathode, 2 – deuterium, 3 – copper anode

A scintillation probe at a distance of 2 m from the chamber has been used to carry out time resolved measurements of the hard X-ray and neutron emission. For each shot the neutron peak from scintillation probe has been recognized and then integrated over the time. The BC-K counter has been triggered manually and the number of counts has been recorded for each shot (counting time of 4 s). Then it has been converted to the total neutron yield. The neutron peak integral versus the total neutron yield measured by means of BC-K for several PF-6 shots is plotted in Fig. 5. Good correlation between two neutron diagnostics can be noticed.

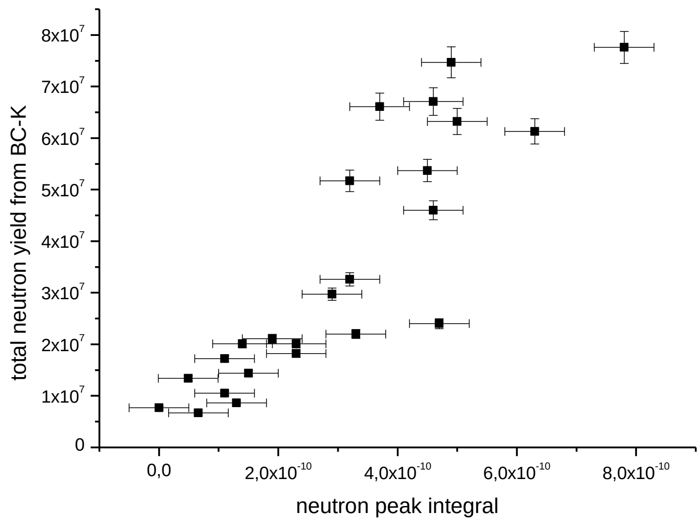

Fig. 5. Neutron peak integral from the scintillation probe versus the total neutron yield measured by means of BC-K.

4.2. Measurements with BC-W at the PF-1000 device

PF-1000 plasma focus operated at IPPLM, Warsaw, is equipped with a 1.332 mF condenser bank and a large volume vacuum chamber (4 m^3). The charging voltage has been 24 kV and the deuterium pressure 0.9 - 2.2 Tr. The BC-W counter has been placed 2.03 m from the anode end at an angle of 175º in respect to the device axis. Calibrated silver activation counters (SAC) have been also used as a standard total neutron yield monitor at PF-1000.

Existing MCNP geometry of PF-1000 device is shown in Fig. 6. BC-W cannot be seen on the picture because it is too small comparing to the device. A point D-D fusion neutron source at a distance of 5 cm from the anode end has been modeled.

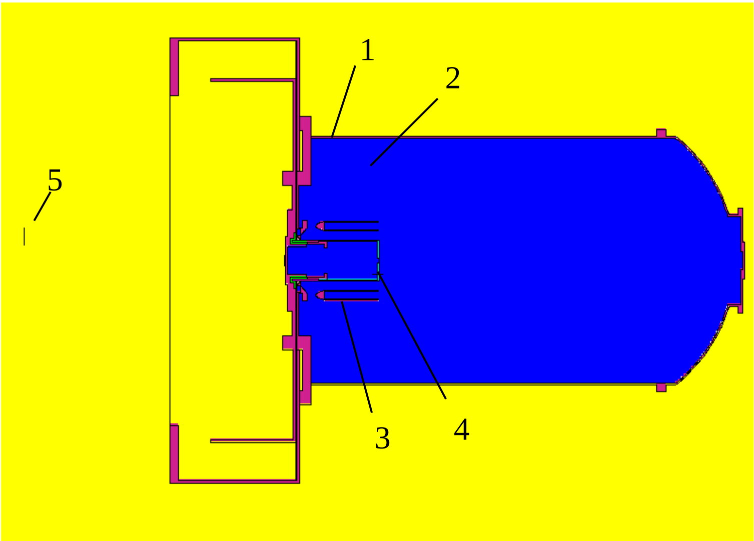

Fig. 6. MCNP geometry for PF-1000 device:
1 – stainless steel vacuum chamber, 2 – deuterium, 3 – stainless steel cathode, 4 – copper anode, 5 – BC-W

The number of counts from SAC and BC-W have been recorded for many PF-1000 shots and converted to total neutron yield using calibration coefficients. Results of those measurements are presented in Fig. 7.

A good correlation between two diagnostics indications can be noticed, however SAC indicate a 3-4 times larger total neutron yield than that of BC-W. It can be explained by a slight displacement of SAC, which happened after last calibration procedure done in 2005, and thus the calibration coefficients for SAC are uncertain.

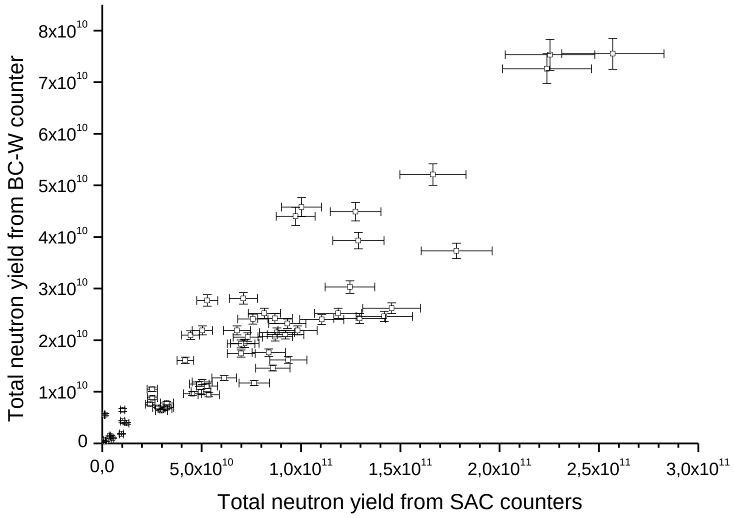

Fig. 7. Total neutron yield from BC-W versus total neutron yield from SAC

4.3. Measurements with BC-K at the PF-4 device

PF-4 plasma focus to be operated in IFJ PAN, Kraków is equipped with a 20 μF condenser bank and a cylindrical vacuum chamber (∅ 219 mm, H = 300 mm). BC-K is designed to operate as a standard neutron monitor at PF-4. An MCNP model of PF-4 with BC-K has been prepared in detail (Fig. 8). Calculations have been carried out for the D-D fusion neutron point source located at various positions on the device axis.

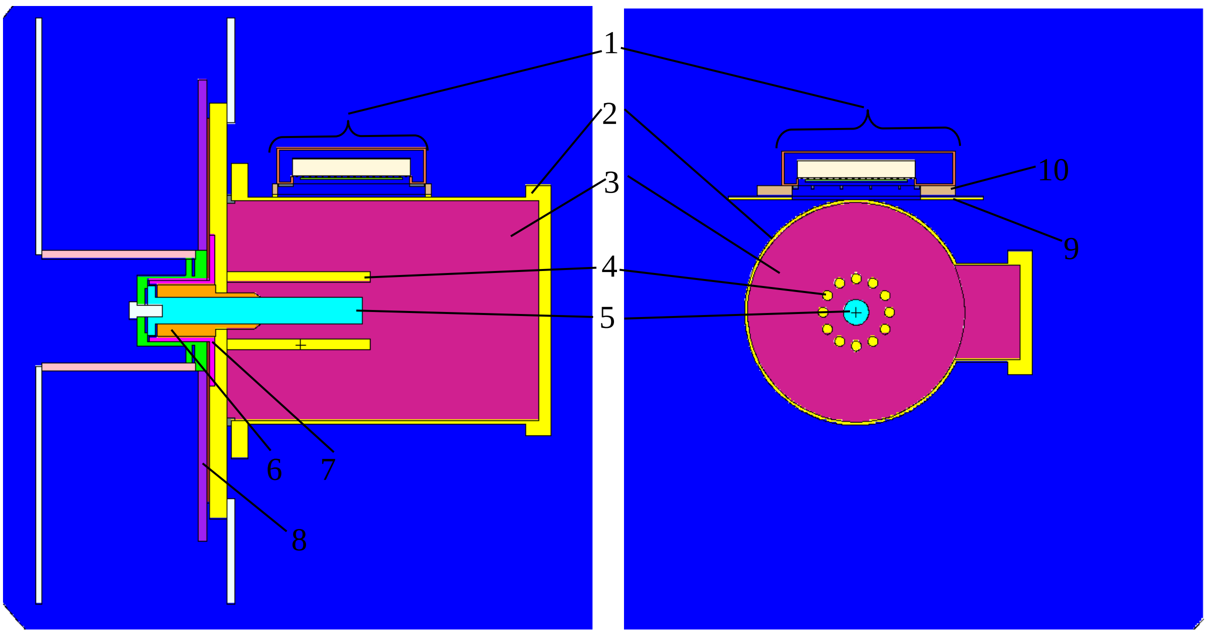

Fig. 8. MCNP geometrical model of PF-4 device with BC-K counter at the top:
1 – BC-K counter, 2 – stainless steel vacuum chamber, 3 – deuterium, 4 – stainless steel outer electrode, 5 – copper inner electrode, 6 – ceramic insulator, 7 – nylon insulator, 8 – high voltage supply cables, 9 – stainless steel pad, 10 – polyamide insulating pad

In order to check the correctness of the BC-K calibration when operated with the PF-4 device the chamber lid has been taken away and a Pu-Be neutron source with nominal yield of $5.03 \cdot 10^5$ s^{-1} has been put inside the chamber in the place where plasma pinch is expected to form. Due to various kinds of radiation emitted by the Pu-Be source it has appeared difficult to distinguish the counts coming from β particles induced by the neutrons (the proportional counter is sensitive to the γ rays and neutrons from the Pu-Be source). Therefore, 2 cm thick bismuth shield has been used to attenuate the γ rays while fast neutrons are going through it uninterrupted. In spite of that, the number of counts coming from ^6He β-decay recorded by the proportional detector has been much lower than the counts caused by γ and neutron radiation of the Pu-Be source. Therefore, the measurements of pulse counting rate with and without beryllium plate has been performed. The difference between results of such two experiments shows the number of β particles recorded by the proportional counter.

The opened vacuum chamber with the Pu-Be source and bismuth shield has been reconstructed in the MCNP calculations (see Fig. 9). The cylindrical volume source of neutrons (in the place where the Pu-Be source has been put) with the energy spectrum taken from [4] has been modeled.

The measurements together with the MCNP calculations allowed determining the yield of used neutron source to $7.4 \cdot 10^5 \pm 2.1 \cdot 10^5$ s^{-1}.

Simultaneously, indium samples placed inside the PF-4 chamber have been irradiated by the Pu-Be calibration source. The MCNP simulation of such activation compared with the samples activity allows verifying neutron yield of the calibration source. As a result of the activation measurements the Pu-Be source neutron yield has been estimated to $6.8 \cdot 10^5 \pm 1.0 \cdot 10^5$ s^{-1}.

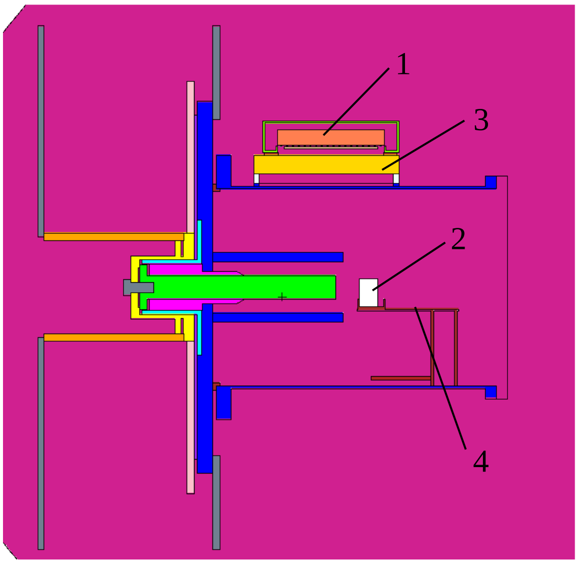

Fig. 9. MCNP geometry of PF-4 with BC-K (1), Pu-Be source (2), bismuth shield (3) and duralumin source holder (4)

5. Conclusions and final remarks

The BC-W designed to be operated on PF-1000 has proved its utility during a number of experiments with results well correlated with SAC. The discrepancy between those two systems arouses due to invalid calibration coefficients of the SAC. It is a recommendation on performing new calibration of SAC at PF-1000.

Before installation of the BC-K on PF-4 it has been tested at the PF-6 device where results have been well correlated with the response of applied scintillation probe.

In spite of the fact that beryllium counter has been designed to pulse neutron source measurements, an attempt to estimate the yield of neutron calibration source by means of BC-K at PF-6 has been made. It has allowed checking the correctness of the MCNP calculations. The obtained result is within the experimental uncertainties.

During further improvements of the diagnostic based on the beryllium counter it has to be noticed that neutron yield should not be determined directly through the number of counts. Due to decay law the exponential curve can be fitted to the time distribution of recorded number of counts. Its exponent is determined by ^{6}He decay constant. The integral over the time of the fitted decay curve is the best estimator of a number of radioactive decays. The fitted curve allows determining the initial activity of the beryllium plate which is also directly proportional to the flux of neutrons causing the reactions.

It is necessary to emphasize that in case of any changes of the geometry of the experimental set-up, further MCNP simulations resulting in new calibration coefficients are needed.

Appendix: β-decay energy spectrum

The β^- particle kinetic energy spectrum implemented to the MCNP calculations was based on the Fermi theory of β-decay [5]. It means that the spectrum of electrons coming from β-decay can be expressed as follows:

$$n(E_e)dE_e = \sqrt{E_e^{\ 2} - (m_e c^2)^2} \cdot E_e \cdot (E_{\max} - E_e)^2 \cdot F(Z, E_e)dE_e, \qquad (6)$$

where $E_e = E_k + m_e c^2$ is the total electron energy being the sum of its kinetic and rest energies; m_e is the electron mass ($m_e c^2 = 0.511$ MeV); E_{max} is the maximal electron energy allowed during the decay; $F(Z,E_e)$ is the Fermi function being an energy correction due to attraction (for β^- decay) or repulsion (for β^+ decay) of the electron or positron leaving the decaying nucleus. The Fermi function for non-relativistic cases can be approximated as follows:

$$F(Z, E_e) = \frac{2\pi\eta}{1 - \exp[-2\pi\eta]} \qquad (7)$$

with

$$\eta = \pm \frac{\alpha}{2\pi} \cdot \frac{Z}{(v/c)}, \qquad (8)$$

where $\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \approx \frac{1}{137}$ is the fine structure constant; Z is atomic number of decay product; v/c is the ratio of the particle to light speeds. The positive sign holds for electrons, the negative for positrons. The relative particle speed is connected with its energy by the following formula:

$$v/c = \sqrt{1 - \frac{1}{(1 + E_e/m_e c^2)^2}}. \qquad (9)$$

Therefore, the β-particle energy spectrum is unequivocally determined by the maximal particle energy and the atomic number of the decay product.

References

[1] G.F. Knoll, *Radiation Detection and Measurements*, Wiley 2000

[2] X-5 Monte Carlo Team, *MCNP – A general Monte Carlo N – Particle Transport Code, Version 5*, LANL, 2003

[3] *MCNP5DATA: Standard Neutron Photoatomic, Photonuclear, and Electron Data Libraries for MCNP5* (CCC-710)

[4] L. Stewart, *Neutron Spectrum and Absolute Yield of a Plutonium-Beryllium Source*, Phys. Rev **98**, (1955)
[5] W.N. Cottingham, D.A. Greenwood, *An Introduction to Nuclear Physics*, Cambridge University Press, 2001

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